Conditional likelihood for use-of-force

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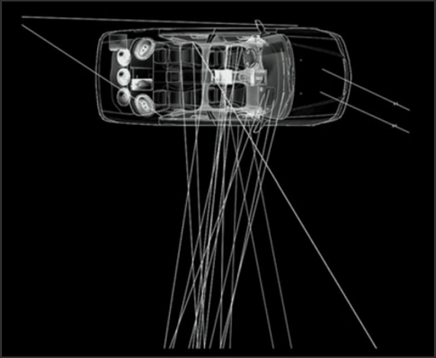
1. Risk of shooting
2. Number of rounds
3. Individual officers escalating

Laquan McDonald shooting, October 20, 2014

- CPD Officer Van Dyke fired 16 rounds
- Officer Walsh fired no rounds, holstering his firearm
- Van Dyke sentenced in January 2019 to 7 years in prison for murder
- Walsh found not guilty of conspiracy to obstruct, left CPD

Sean Bell shooting, November 25, 2006



- Detective Oliver, age 35, white, 31 rounds
- Detective Isnora, age 28, black, 11 rounds
- Detective Cooper, age 39, black, 4 rounds
- Officer Carey, age 26, white, 3 rounds
- Detective Headley, age 35, black, 1 round

Force depends on officer and environment

- officer with characteristics $\mathbf{x}$ (e.g., age, race, sex, experience, prior involvement in shootings)
- environment $\mathbf{z}$, shared situational, organizational, community, and legal factors (e.g., time, place, lighting, suspect features, governing policies and laws, community conditions)
- $Y = 1$ indicates the officer discharged firearm, $Y = 0$ otherwise

$$\log \frac{P(Y=1|\mathbf{x},\mathbf{z})}{P(Y=0|\mathbf{x},\mathbf{z})} = \alpha'\mathbf{z} + \beta'\mathbf{x} \tag{1}$$

Logistic regression predicting Y from $\mathbf{z}$ and $\mathbf{x}$

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Logistic regression predicting Y from $\mathbf{z}$ and $\mathbf{x}$

Assignment is a confounder

Assignment is related to *both* $\mathbf{x}$ and $\mathbf{z}$

- "The overrepresentation of minority officers among police shooters [is] closely associated with racially varying pattern of assignment, socialization, and residence" – Fyfe (1981)
- "It is quite possible that other factors, such as the extent to which college-educated officers versus non-college-educated officers encounter resistant suspects, may account for why education appears to matter" – Paoline and Terrill (2007)
- "Based on an officer's rank, time on the job, age, and gender, he or she may have been less active, assigned to areas with lower crime rates, or working in a position that did not have frequent contact with citizens" – McElvain and Kposowa (2008)

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Confounding implies $\hat{\beta}$ sensitive to model structure

$$\log \frac{P(Y=1|\mathbf{x},\mathbf{z})}{P(Y=0|\mathbf{x},\mathbf{z})} = \alpha'\mathbf{z} + \beta'\mathbf{x} \tag{2}$$

- Must measure all important elements in $\mathbf{z}$
- Must correctly capture non-linear effects and interactions
- $\mathbf{z}$ is really expensive and statistically annoying

How we estimate logistic regression

$$\log \frac{P(Y=1|\mathbf{x},\mathbf{z})}{P(Y=0|\mathbf{x},\mathbf{z})} = \alpha'\mathbf{z} + \beta'\mathbf{x} \tag{3}$$

- Find α and β that maximize the log-likelihood function

$$\begin{aligned} \ell(\alpha,\beta) &= \log P(Y_1=y_1|\mathbf{z}_1,\mathbf{x}_1)\cdots P(Y_n=y_n|\mathbf{z}_n,\mathbf{x}_n) && (4) \\ &= \sum_{i=1}^{n} y_i(\alpha'\mathbf{z}+\beta'\mathbf{x}) - \log\left(1+\exp\left(\alpha'\mathbf{z}+\beta'\mathbf{x}\right)\right) && (5) \end{aligned}$$

- Easily optimized with iteratively reweighted least squares (IRLS)

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- Easily optimized with iteratively reweighted least squares (IRLS)

Rethink the likelihood for a sampled moment in time

$$\ell_i(\alpha,\beta) = \log P(Y_1 = y_1, \ldots, Y_n = y_n | \mathbf{x}_1, \ldots, \mathbf{x}_n, \mathbf{z}, \alpha, \beta) \tag{6}$$

- y_j is a 0/1 indicator of whether officer j shoots at this moment
- All officers on scene share the same value of $\mathbf{z}$
- Log-likelihood from all sampled instances is the sum such terms, one for each sampled instance

G. Ridgeway (2016). "Officer Risk Factors Associated with Police Shootings: A Matched Case-Control Study," *Statistics and Public Policy* 3(1):1-6

Condition on a sufficient statistic

- Let $S_i = Y_1 + \ldots + Y_{n_i}$, number of shooters in incident i
- Recall $P(A, B) = P(A|B)P(B)$

$$
\begin{aligned}
L_i(\alpha,\beta) &= P(Y_1 = y_1, \ldots, Y_{n_i} = y_{n_i} | \mathbf{x}_1, \ldots, \mathbf{x}_{n_i}, \mathbf{z}, \alpha, \beta) \\
&= P(Y_1 = y_1, \ldots, Y_{n_i} = y_{n_i}, S_i | \mathbf{x}_1, \ldots, \mathbf{x}_{n_i}, \mathbf{z}, \alpha, \beta) \\
&= P(Y_1 = y_1, \ldots, Y_{n_i} = y_{n_i} | S_i, \mathbf{x}_1, \ldots, \mathbf{x}_{n_i}, \mathbf{z}, \alpha, \beta) \times \\
&\qquad P(S_i | \mathbf{x}_1, \ldots, \mathbf{x}_{n_i}, \mathbf{z}, \alpha, \beta) \\
&= \text{individual officer likelihood} \times \\
&\qquad \text{collective group likelihood}
\end{aligned}
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- Manski & Lerman (1977) and Prentice & Pyke (1979) showed that $\hat{\beta}$ using any or all of these terms are consistent for β (but not β_0)

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Individual officer likelihood simplifies

- Recall Bayes Theorem $P(A|B) = P(B|A)P(A)/P(B)$
- Consider a moment with two officers

$$
\begin{aligned}
&P(Y_1 = 1, Y_2 = 0 | S = 1, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \\
&= \frac{P(S = 1 | Y_1 = 1, Y_2 = 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) P(Y_1 = 1, Y_2 = 0 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{z})}{P(S = 1 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{z})} \\
&= \frac{1 \times P(Y_1 = 1 | \mathbf{x}_1, \mathbf{z}) P(Y_2 = 0 | \mathbf{x}_2, \mathbf{z})}{P(S = 1 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{z})} \\
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&= \frac{\frac{e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_1}}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_1}} \frac{1}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_2}}}{\frac{1}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_1}} \frac{e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_2}}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_2}} + \frac{e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_1}}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_1}} \frac{1}{1+e^{\alpha'\mathbf{z}+\beta'\mathbf{x}_2}}}
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&P(Y_1 = 1, Y_2 = 0|S = 1, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \\
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= & \frac{e^{\beta' \mathbf{x}_1}}{e^{\beta' \mathbf{x}_1} + e^{\beta' \mathbf{x}_2}}
\end{aligned}
$$

An example with four officers

$$P(Y_1 = 1, Y_2 = 0, Y_3 = 1, Y_4 = 0 | S = 2, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{z})$$

$$= \frac{e^{\beta'\mathbf{x}_1} e^{\beta'\mathbf{x}_3}}{e^{\beta'\mathbf{x}_1} e^{\beta'\mathbf{x}_2} + e^{\beta'\mathbf{x}_1} e^{\beta'\mathbf{x}_3} + e^{\beta'\mathbf{x}_1} e^{\beta'\mathbf{x}_4} + e^{\beta'\mathbf{x}_2} e^{\beta'\mathbf{x}_3} + e^{\beta'\mathbf{x}_2} e^{\beta'\mathbf{x}_4} + e^{\beta'\mathbf{x}_3} e^{\beta'\mathbf{x}_4}}$$

An example with four officers

If all four officers had discharged their firearm, then

$$P(Y_1 = 1, Y_2 = 1, Y_3 = 1, Y_4 = 1 | S = 4, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{z})$$
$$= \frac{e^{\beta' \mathbf{x}_1} e^{\beta' \mathbf{x}_2} e^{\beta' \mathbf{x}_3} e^{\beta' \mathbf{x}_4}}{e^{\beta' \mathbf{x}_1} e^{\beta' \mathbf{x}_2} e^{\beta' \mathbf{x}_3} e^{\beta' \mathbf{x}_4}} = 1$$

An example with four officers

If none of the officers had discharged their firearm, then

$$P(Y_1 = 0, Y_2 = 0, Y_3 = 0, Y_4 = 0 | S = 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{z}) = \frac{1 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 1} = 1$$

- Moments in which no officers shoot provide no information about β
- Moments with everyone shooting provide no information about β

The only moments and places with information on β are shootings with multiple officers where not all officers shoot

An example with four officers

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$$
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NYPD analysis, 239 shooters, 155 non-shooters, 175 incidents

Variable	Odds-ratio	95% interval	Permutation p-value
Rank			
Police officer (reference)			
Detective	1.2	(0.2,6.3)	0.78
Sergeant	*0.2	(0.1,0.7)	0.006
Lieutenant	*0.0	(0.0,0.4)	0.003
Captain	0.1	(0.0,0.8)	0.16
Years at NYPD	1.0	(0.9,1.1)	0.89
Age when recruited	*0.9	(0.8,1.0)	0.03
Race			
White (reference)			
Black	*3.3	(1.2,8.9)	0.01
Other	1.2	(0.5,2.8)	0.71
Male	2.1	(0.5,8.9)	0.29
Education			
High school (reference)			
High school+some college	1.3	(0.5,3.0)	0.60
College	1.9	(0.6,6.1)	0.26
College+some graduate	1.8	(0.1,22.7)	0.68
Precinct officer	0.2	(0.0,1.2)	0.08

NYPD analysis, 239 shooters, 155 non-shooters, 175 incidents

Variable	Odds-ratio	95% interval	Permutation p-value
Rank			
Police officer (reference)			
Detective	1.2	(0.2,6.3)	0.78
Sergeant	*0.2	(0.1,0.7)	0.006
Lieutenant	*0.0	(0.0,0.4)	0.003
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NYPD - 239 shooters, 155 non-shooters, 175 incidents

Variable	Odds-ratio	95% interval	Permutation p-value
Average annual			
Evaluation score < 3.5	0.7	(0.3,1.8)	0.45
Range score < 86	1.8	(0.7,4.5)	0.16
Complaints > 0.6	2.1	(0.7,6.4)	0.15
Medal count/year > 3.8	2.3	(0.5,9.5)	0.21
CPI points > 3.1	*3.1	(1.0,8.9)	0.03
Gun arrests > 2.4	0.7	(0.2,2.5)	0.64
Felony arrests > 9.3	2.1	(0.6,7.0)	0.20
Misdemeanor arrests > 10.0	*0.2	(0.1,0.6)	0.002
Days of leave			
Not due to line of duty injury > 8.4	0.9	(0.4,2.1)	0.81
Due to line of duty injury > 5.6	0.9	(0.3,2.4)	0.82

Major Cities Chiefs (MCCA) and Police Foundation Standardized Collection

- 56 agencies in the U.S. and Canada contributed data
- Full dataset describes 2,574 officers involved in 1,600 shootings between 2010-2018
- 317 multi-officer shootings, 849 officers, 5,026 rounds
- However, only included data on officers who discharged their firearm

G. Ridgeway, B. Cave, J. Grieco, and C.E. Loeffler (2021). "A Conditional Likelihood Model of the Relationship Between Officer Features and Rounds Discharged in Police Shootings," *Journal of Quantitative Criminology* 37(1):303-326.

Conditional likelihood for rounds fired

Assume that $R_i \sim \mathrm{Poisson}(\lambda_i)$ where $\log(\lambda_i) = \alpha'\mathbf{z} + \beta'\mathbf{x}_i$.

$$
\begin{aligned}
& P(R_1 = 3, R_2 = 4 | R_1 + R_2 = 7, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \\
& \quad = \binom{7}{3} \left(\frac{e^{\beta'\mathbf{x}_1}}{e^{\beta'\mathbf{x}_1} + e^{\beta'\mathbf{x}_2}} \right)^3 \left(1 - \frac{e^{\beta'\mathbf{x}_1}}{e^{\beta'\mathbf{x}_1} + e^{\beta'\mathbf{x}_2}} \right)^4
\end{aligned}
$$

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No witness officers included

- Data collected did not include officers on the scene who did not shoot
- Everything is conditional on $R > 0$
- Replace Poisson with a truncated Poisson

$$P(R_1 = r | R_1 > 0, \mathbf{x}_1, \mathbf{z}) \quad = \quad \frac{\lambda_1^r e^{-\lambda_1}}{r!(1 - e^{-\lambda_1})}, r > 0$$

$$P(R_1 = 3, R_2 = 4 | R_1 + R_2 = 7, R_1 > 0, R_2 > 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z})$$
$$= \quad \frac{(e^{\beta' \mathbf{x}_1})^3}{3!} \frac{(e^{\beta' \mathbf{x}_2})^4}{4!} \frac{1}{\sum_{r_1 + r_2 = 7, r_1 > 0, r_2 > 0} \frac{(e^{\beta' \mathbf{x}_1})^{r_1}}{r_1!} \frac{(e^{\beta' \mathbf{x}_2})^{r_2}}{r_2!}}$$

Recursion or Monte Carlo estimation of denominator

- Most complex shooting involved 11 officers discharging 88 rounds, 4,000,751,045,226 terms
- Efficient recursion algorithm computes denominator for second most complex shooting (8 officers, 58 rounds) in 20s
- Most complex shooting has 15,000$\times$ more terms and would take 3 days
- Optimization requires hundreds of likelihood calculations
- Sum can be rewritten to resemble $E\left(\frac{1}{(r_1+1)(r_2+1)\ldots(r_n+1)}\right)$ where $r_1, r_2, \ldots, r_n$ are a drawn from a multinomial distribution (5 seconds)

46 agencies, 317 shootings, 849 officers, 5026 rounds

Variable	Rate ratio	95% interval	Permutation p-value
Age at recruitment	1.01	(0.99,1.02)	0.31
Years of experience	1.00	(0.98,1.01)	0.58
Female	0.86	(0.64,1.14)	0.27
Race (relative to white)			
Black	1.05	(0.86,1.28)	0.64
Hispanic	1.09	(0.89,1.32)	0.39
Other	0.76	(0.57,1.01)	0.05
Prior OIS (relative to 0)			
1 or more	1.02	(0.84,1.24)	0.85
2 or more	1.23	(0.88,1.72)	0.21
Prior force complaint	1.25	(0.92,1.69)	0.14
Role			
Detective	1.09	(0.78,1.52)	0.61
Sergeant or more senior	1.03	(0.87,1.22)	0.75
Other	0.66	(0.32,1.37)	0.26
Special assignment	1.28	(0.97,1.68)	0.07
Long gun (relative to pistol)	1.01	(0.78,1.30)	0.97

Ordered stereotype model for force escalation

- Expand beyond shootings to other modes of force
- Identify individual officer effects, not officer features
- Let $Y = 0, \ldots, K$ order severity of use-of-force from $Y = 0$ representing no force to $Y = K$ representing lethal force

$$P(Y = y|\mathbf{z}, \mathbf{x}) = \frac{\exp(\theta_y + s_y(\alpha'\mathbf{z} + \beta'\mathbf{x}))}{\sum_{k=1}^{K} \exp(\theta_k + s_k(\alpha'\mathbf{z} + \beta'\mathbf{x}))}$$

- $\theta_0 = 0$, $s_0 = 0$, $s_1 = 1$ for identifiability
- s_k effectively quantify the "distance" between force levels

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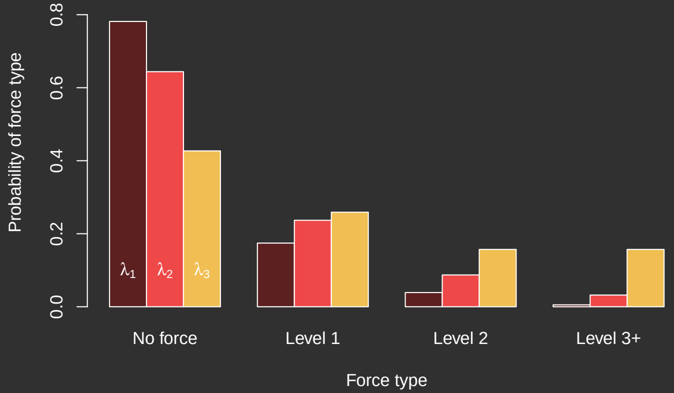
Which specific officers most likely to escalate?

$$P(Y_j = y|\mathbf{z}) \quad = \quad \frac{\exp(\theta_y + s_y(\gamma'\mathbf{z} + \lambda_j))}{\sum_{k=1}^{K} \exp(\theta_k + s_k(\gamma'\mathbf{z} + \lambda_j))}$$

Ordered stereotype can model force type selection

- $\theta = \{0, -1, -2, -3\}$
- $s = \{0, 1, 2, 4\}$
- $\lambda_1 = -\frac{1}{2},\ \lambda_2 = 0,\ \lambda_3 = \frac{1}{2}$



Conditional likelihood for ordered stereotype model

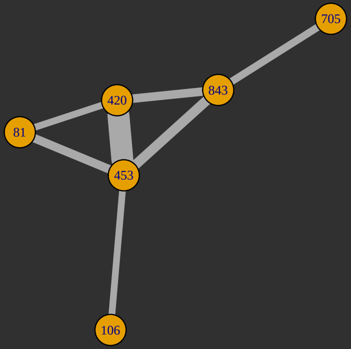
- First officer does nothing, $Y_1 = 0$
- Second officer holds and pushes, $Y_2 = 1$
- Third officer strikes with a baton, $Y_3 = 3$

Conditional likelihood terms look like

$$P(Y_1 = 0, Y_2 = 1, Y_3 = 3) = \frac{e^{s_0\lambda_1+s_1\lambda_2+s_3\lambda_3}}{e^{s_0\lambda_1+s_1\lambda_2+s_3\lambda_3} + \ldots + e^{s_3\lambda_1+s_1\lambda_2+s_0\lambda_3}}$$

Seattle PD data

- 1,386 unique officers, 635 appear have fewer than 10 force incidents
- 3,701 force incidents with information
  - More than one officer
  - Variation in force type used
- An example subnetwork

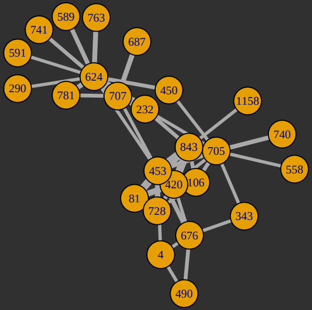


Estimated λs

Officer ID	λ	% Max force
81	-0.22	0.33
106	-0.21	0.30
453	-0.19	0.36
420	0.11	0.43
705	0.11	0.33
843	0.40	0.53

- Identification is hardest for least connected nodes (106 & 705)

Expanded network allows better precision for some officers



ID	λ	% Max force
4	-2.18	0.00
232	-1.93	0.00
290	-0.78	0.15
763	-0.51	0.15
740	-0.47	0.14
450	-0.46	0.14
705	-0.45	0.15
343	-0.34	0.26
676	-0.29	0.27
81	-0.07	0.27
781	-0.04	0.18
1158	-0.03	0.25
420	0.02	0.33
589	0.04	0.22
453	0.06	0.31
106	0.17	0.33
728	0.45	0.45
687	0.46	0.38
624	0.47	0.36
490	0.50	0.50
843	0.65	0.45
741	0.79	0.50
591	0.81	0.44
558	1.00	0.60
707	1.46	0.69

Conclusion

- Conditional likelihood solves the long-standing problem of confounding by assignment
- The approach has potential beyond shooting decisions to force severity
- May be useful as a use-of-force early warning system
- Interesting combination of policing, statistics, mathematics, and computer science
- Opportunities
  - apply to new departments
  - assess other police performance measures
  - check robustness to contagion/anti-contagion

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Conditional likelihood for use-of-force

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